

Critical properties of the N -color London model

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The critical properties of N -color London model are studied in $d = 2 + 1$ dimensions. The model is dualized to a theory of N vortex fields interacting through a Coulomb and a screened potential. The model with $N = 2$ shows two anomalies in the specific heat. From the critical exponents α and ν , the mass of the gauge field, and the vortex correlation functions, we conclude that one anomaly corresponds to an *inverted* 3Dxy fixed point, while the other corresponds to a 3Dxy fixed point. There are N fixed points, namely one corresponding to an inverted 3Dxy fixed point, and $N - 1$ corresponding to neutral 3Dxy fixed points. This represents a novel type of quantum fluid, where superfluid modes arise out of charged condensates.

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Ginzburg-Landau (GL) theories with several complex scalar matter fields minimally coupled to one gauge field are of interest in a wide variety of systems, such as multiple component (color) superconductors, metallic phases of light atoms such as hydrogen [1, 2], and as effective theories for easy-plane quantum antiferromagnets [3, 4, 5]. The model also is highly relevant in particle physics where it is called two-Higgs doublet model [6]. In metallic hydrogen the scalar fields represent Cooper pairs of electrons and protons, which excludes the possibility of inter-color pair tunneling, i.e. there is no Josephson coupling between different components of the condensate. The same two-color action in $(2 + 1)$ dimensions, where the matter fields originate in a bosonic representation of spin operators, is claimed to be the critical sector of a field theory separating a Néel state and a paramagnetic (valence bond ordered) state of a two dimensional quantum antiferromagnet at zero temperature with an easy-plane anisotropy present [3, 5]. This happens because, although the effective description of the antiferromagnet involves an *a priori* compact gauge field, it must be supplemented by Berry-phase terms in order to properly describe $S = 1/2$ spin systems [7, 8]. Berry-phases cancel the effects of monopoles at the critical point [3, 5]. In this paper, we point out novel physics of the quantum fluid that arises out of an N -color charged condensate when no intercolor Josephson coupling is present.

For a detailed analysis of the phase transitions in such a generalized GL model, we study an N component GL theory in $(2 + 1)$ dimensions with no Josephson coupling term. The model is defined by N complex scalar fields $\{\Psi^{(\alpha)}(\mathbf{r}) \mid \alpha = 1 \dots N\}$ coupled through the charge e to a fluctuating gauge field $\mathbf{A}(\mathbf{r})$, with Hamiltonian

$$H = \sum_{\alpha=1}^N \frac{|\nabla - ie\mathbf{A}\Psi^{(\alpha)}|^2}{2M^{(\alpha)}} + V(\{\Psi^{(\alpha)}\}) + \frac{1}{2}(\nabla \times \mathbf{A})^2 \quad (1)$$

where $M^{(\alpha)}$ is the α -component condensate mass. The potential $V(\{\Psi^{(\alpha)}(\mathbf{r})\})$ is assumed to be only a function of $|\Psi^{(\alpha)}(\mathbf{r})|^2$. The model is studied in the phase-only

(London) approximation $\Psi^{(\alpha)}(\mathbf{r}) = |\Psi_0^{(\alpha)}| \exp[i\theta^{(\alpha)}(\mathbf{r})]$ and is discretized on a lattice with spacing $a = 1$ [9]. In the Villain approximation the partition function reads

$$Z = \int_{-\infty}^{\infty} \mathcal{D}\mathbf{A} \prod_{\gamma=1}^N \int_{-\pi}^{\pi} \mathcal{D}\theta^{(\gamma)} \prod_{\eta=1}^N \sum_{\mathbf{n}^{(\eta)}} \exp(-S) \\ S = \sum_{\mathbf{r}} \left(\sum_{\alpha=1}^N \frac{\beta |\Psi_0^{(\alpha)}|^2}{2M^{(\alpha)}} (\Delta\theta^{(\alpha)} - e\mathbf{A} + 2\pi\mathbf{n}^{(\alpha)})^2 \right. \\ \left. + \frac{\beta}{2} (\Delta \times \mathbf{A})^2 \right), \quad (2)$$

where $\mathbf{n}^{(\alpha)}(\mathbf{r})$ are integer vector fields ensuring 2π periodicity, and the lattice position index vector \mathbf{r} of the fields is suppressed. The symbol Δ denotes the lattice difference operator and $\beta = 1/T$ is the inverse temperature. *Here, we stress the importance of keeping track of the 2π periodicity of the individual phases.* The kinetic energy terms are linearized by introducing N auxiliary fields $\mathbf{v}^{(\alpha)}$. Integration over all $\theta^{(\alpha)}$ produces the local constraints $\Delta \cdot \mathbf{v}^{(\alpha)} = 0$, which are fulfilled by the replacement $\mathbf{v}^{(\alpha)} \rightarrow \Delta \times \mathbf{h}^{(\alpha)}$. We recognize $\mathbf{h}^{(\alpha)}$ as the dual gauge fields of the theory. By fixing the gauge $n_z^{(\alpha)} = 0$ and performing a partial integration we may introduce the vortex fields $\mathbf{m}^{(\alpha)} = \Delta \times \mathbf{n}^{(\alpha)}$. We integrate out the gauge field \mathbf{A} and get a theory in the dual gauge fields $\mathbf{h}^{(\alpha)}$ and the vortex fields $\mathbf{m}^{(\alpha)}$ where $\Delta \cdot \mathbf{m}^{(\alpha)} = 0$

$$S = \sum_{\mathbf{r}} \left[2\pi i \sum_{\alpha=1}^N \mathbf{m}^{(\alpha)} \cdot \mathbf{h}^{(\alpha)} + \sum_{\alpha=1}^N \frac{(\Delta \times \mathbf{h}^{(\alpha)})^2}{2\beta |\psi^{(\alpha)}|^2} \right. \\ \left. + \frac{e^2}{2\beta} \left(\sum_{\alpha=1}^N \mathbf{h}^{(\alpha)} \right)^2 \right], \quad (3)$$

where $|\psi^{(\alpha)}|^2 = |\Psi_0^{(\alpha)}|^2/M^{(\alpha)}$. Note how the *algebraic sum* of the dual photon fields is massive. This differs from the case $N = 1$, where e produces one massive dual photon with bare mass $e^2/2$, and the model describes a

vortex field \mathbf{m} interacting through a *massive* dual gauge field \mathbf{h} . However, when $N \geq 2$, since $\Delta \cdot \mathbf{m}^{(\alpha)} = 0$, a gauge transformation $\mathbf{h}^{(\alpha)} \rightarrow \mathbf{h}^{(\alpha)} + \Delta g^{(\alpha)}$ for $\alpha = 1 \dots N$ leaves the action invariant if one of the gauge fields, say $\mathbf{h}^{(\eta)}$ compensates the sum in the last term in (3) with $\Delta g^{(\eta)} = -\sum_{\alpha \neq \eta} \Delta g^{(\alpha)}$.

Integrating out the dual gauge fields we get a generalized theory of N interacting vortex fields

$$Z = \sum_{\mathbf{m}^{(1)}} \cdots \sum_{\mathbf{m}^{(N)}} \delta_{\Delta \cdot \mathbf{m}^{(1)}, 0} \cdots \delta_{\Delta \cdot \mathbf{m}^{(N)}, 0} \times e^{-S_V} \quad (4)$$

$$S_V = \sum_{\mathbf{r}, \mathbf{r}'} \sum_{\alpha, \eta} \mathbf{m}^{(\alpha)}(\mathbf{r}) D^{(\alpha, \eta)}(\mathbf{r} - \mathbf{r}') \mathbf{m}^{(\eta)}(\mathbf{r}')$$

where $\delta_{x,y}$ is the Kronecker-delta, and the vortex interaction potential $D^{(\alpha, \eta)}(\mathbf{r})$ is the inverse discrete Fourier transform of $\tilde{D}^{(\alpha, \eta)}(\mathbf{q})$, where

$$\frac{\tilde{D}^{(\alpha, \eta)}(\mathbf{q})}{2\pi^2 \beta |\psi^{(\alpha)}|^2} = \frac{\lambda^{(\eta)}}{|\mathbf{Q}_q|^2 + m_0^2} + \frac{\delta_{\alpha, \eta} - \lambda^{(\eta)}}{|\mathbf{Q}_q|^2}, \quad (5)$$

$\lambda^{(\alpha)} = |\psi^{(\alpha)}|^2 / \psi^2$ and $\psi^2 = \sum_{\alpha=1}^N |\psi^{(\alpha)}|^2$. Here, $m_0^2 = e^2 \psi^2$ is the square of the bare inverse screening length in the intervortex interaction, and $|\mathbf{Q}_q|^2$ is the Fourier representation of the lattice Laplace operator. The first term of the vortex interaction potential (5) is a Yukawa screened potential, *while the second term mediates long range Coulomb interaction between vortex fields*. If $N = 1$ the latter cancels out exactly and we are left with the well studied vortex theory of the GL model which has a charged fixed point for $e \neq 0$ [10, 11]. For $N \geq 2$ we find a theory of vortex loops of N colors interacting through long range Coulomb interaction. If N grows to infinity, $\psi^2 \rightarrow \infty$ and the vortex fields interact via a diagonal unscreened $N \times N$ Coulomb matrix. This reflects the inability of one single gauge field \mathbf{A} to screen a large number of vortex species. *The case $N \geq 2$ has features with no counterpart in the case $N = 1$ [9, 11], namely neutral superfluid modes arising out of charged condensates.*

The above vortex system may be formulated as a field theory, introducing N complex matter fields $\phi^{(\alpha)}$ for each vortex species, minimally coupled to the dual gauge fields $\mathbf{h}^{(\alpha)}$. This generalizes the dual theory for $N = 1$ pioneered in [13]. The theory reads (see also [5])

$$S_{\text{dual}} = \sum_{\mathbf{r}} \left[\sum_{\alpha=1}^N \left(m_{\alpha}^2 |\phi^{(\alpha)}|^2 + |(\Delta - i\mathbf{h}^{(\alpha)})\phi^{(\alpha)}|^2 \right. \right. \\ \left. \left. + \frac{(\Delta \times \mathbf{h}^{(\alpha)})^2}{2\beta |\psi^{(\alpha)}|^2} \right) + \frac{e^2}{2\beta} \left(\sum_{\alpha=1}^N \mathbf{h}^{(\alpha)} \right)^2 \right. \\ \left. + \sum_{\alpha, \eta} g^{(\alpha, \eta)} |\phi^{(\alpha)}|^2 |\phi^{(\eta)}|^2 \right]. \quad (6)$$

Here, we have added chemical potential (core-energy) terms for the vortices, as well as steric short-range repulsion interactions between vortex elements. In the $N = 1$

case, a RG treatment of the mass term of the dual gauge field yields $\partial e^2 / \partial \ln l = e^2$, and hence this term scales up, suppressing the dual gauge field. Correspondingly, for $N \geq 2$, this suppresses $\sum_{\alpha} \mathbf{h}^{(\alpha)}$, but not each individual dual gauge field. For the particular case $N = 2$, assuming the same to hold, we end up with a gauge theory of two complex matter fields coupled minimally to one massless gauge field, which was also precisely the starting point. Thus the theory is self-dual for $N = 2$ [4, 5]. For $N = 1$, it is known that a charged theory in $d = 2 + 1$ dualizes into a $|\phi|^4$ theory and vice versa [11]. The vortex tangle of the 3Dxy model is incompressible and the dual theory is a massless gauge theory such that $\langle \phi \rangle \neq 0$ is prohibited. For $e \neq 0$, the dual theory has global symmetry, and vortex condensation and $\langle \phi \rangle \neq 0$ is possible [11].

For $N = 2$, Monte Carlo (MC) simulations have been carried out for the action (4) with parameters $|\psi^{(1)}|^2 = 1/2$, $|\psi^{(2)}|^2 = 1$, $e^2 = 1/4$, and $m_0^2 = 3/8$. Here, $|\psi^{(1)}|^2$ and $|\psi^{(2)}|^2$ have been chosen to have well-separated bare energy scales associated with the twist of the two types of phases, and m_0 has been chosen to be of the order of the inverse lattice spacing in the problem to avoid difficult finite-size effects. One MC update consists of inserting elementary vortex loops of random direction and species according to the Metropolis algorithm.

We observe two anomalies in the specific heat at T_{c1} and T_{c2} where $T_{c1} < T_{c2}$. We find T_{c1} and T_{c2} from scaling of the second moment of the action $\langle (S_V - \langle S_V \rangle)^2 \rangle$ to be $T_{c1} = 1.4(6)$ and $T_{c2} = 2.7(8)$. To check the criticality of these anomalies we have calculated the critical exponents α and ν by applying finite size scaling (FSS) of $M_3 = \langle (S_V - \langle S_V \rangle)^3 \rangle$ [14]. The peak to peak value of this quantity scales with system size L as $L^{(1+\alpha)/\nu}$, the width between the peaks scales as $L^{-1/\nu}$. The advantage of this is that asymptotically correct behavior is reached for practical system sizes. The FSS plots for system sizes $L = 4, 6, 8, 10, 12, 14, 16, 20, 24$ are shown in Fig. 1. From the scaling we conclude that both anomalies are in fact critical points, and we obtain $\alpha = -0.02 \pm 0.02$ and $\nu = 0.67 \pm 0.01$ for T_{c1} and $\alpha = -0.03 \pm 0.02$ and $\nu = 0.67 \pm 0.01$ for T_{c2} . These values are consistent with those of the 3Dxy and the *inverted* 3Dxy universality classes found with high precision to be $\alpha = -0.0146(8)$ and $\nu = 0.67155(3)$ [15].

To characterize these phase transitions further, we consider $\mathcal{G}_{\mathbf{A}}(q) = \langle \mathbf{A}_q \cdot \mathbf{A}_{-q} \rangle$ and $\mathcal{G}_{\Sigma \mathbf{h}}(q) = \langle (\sum_{\alpha} \mathbf{h}_q^{(\alpha)}) \cdot (\sum_{\alpha} \mathbf{h}_{-q}^{(\alpha)}) \rangle$, expressed in terms of $G^{(+)}(q) = \langle |\sum_{\alpha} |\psi^{(\alpha)}|^2 \mathbf{m}_q^{(\alpha)}|^2 \rangle$ as

$$\mathcal{G}_{\mathbf{A}}(q) = \frac{2/\beta}{|\mathbf{Q}_q|^2 + m_0^2} \left(1 + \frac{2\pi^2 \beta m_0^2}{|\mathbf{Q}_q|^2} \frac{G^{(+)}(q)}{|\mathbf{Q}_q|^2 + m_0^2} \right) \quad (7)$$

$$\mathcal{G}_{\Sigma \mathbf{h}}(q) = \frac{2\beta \psi^2}{|\mathbf{Q}_q|^2 + m_0^2} \left(1 - \frac{2\pi^2 \beta}{\psi^2} \frac{G^{(+)}(q)}{|\mathbf{Q}_q|^2 + m_0^2} \right).$$

The masses of \mathbf{A} and $\sum_{\alpha} \mathbf{h}^{(\alpha)}$ are defined by $m_{\mathbf{A}}^2 =$

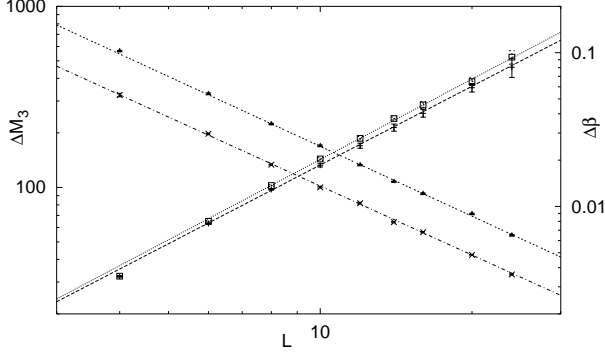


FIG. 1: The FSS of the peak to peak value of the third moment ΔM_3 labeled (\square) and ($+$) for T_{c1} and T_{c2} respectively. The scaling of the width between the peaks $\Delta\beta$ labeled (\blacktriangle) and (\times) for T_{c1} and T_{c2} respectively. The lines are power law fits to the data for $L > 6$ used to extract α and ν .

$$\lim_{q \rightarrow 0} 2\mathcal{G}_A(q)^{-1}/\beta \text{ and } m_{\Sigma h}^2 = \lim_{q \rightarrow 0} 2\beta\psi^2\mathcal{G}_{\Sigma h}(q)^{-1}.$$

We briefly review the case $N = 1$ [11]. The dual field theory of the neutral fixed point ($m_0^2 = 0$) is a charged theory describing an incompressible vortex tangle. The leading behavior of the vortex correlator is $\lim_{q \rightarrow 0} 2\pi^2\beta G^{(+)}(q) \sim [1 - C_2(T)]q^2$, $q^2 - C_3(T)q^{2+\eta_A}$, and $q^2 + C_4(T)q^4$ for $T < T_c$, $T = T_c$, and $T > T_c$ respectively. For $T < T_c$ we have $m_{\Sigma h}^2 = 0$ ($N = 1$), however for $T > T_c$ the $1/q^2$ terms in $\mathcal{G}_{\Sigma h}(q)$ cancel out exactly and this mass attains an expectation value. At the charged fixed point ($m_0^2 \neq 0$) of the GL model, the effective field theory of the vortices is a neutral theory. The vortex tangle is compressible with a scaling ansatz for the vortex correlator $\lim_{q \rightarrow 0} G^{(+)}(q) \sim q^2$, $q^{2-\eta_A}$, and $c(T)$ for $T < T_c$, $T = T_c$, and $T \geq T_c$, respectively. Consequently, from (7), the mass m_A drops to zero at T_c , and the mass of the dual gauge field m_h is finite for all temperatures and has a kink at T_c . Renormalization group arguments yield $\eta_A = 4 - d$ where d is the dimensionality [10, 12], which has recently been verified numerically [11, 16].

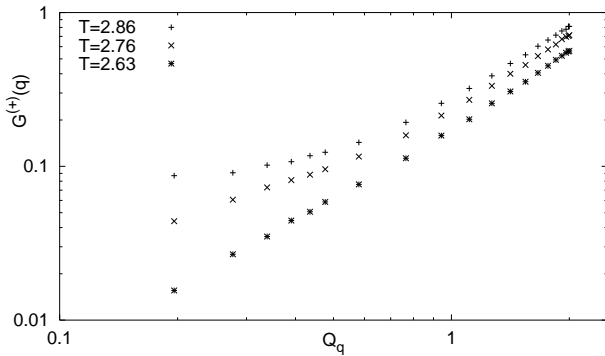


FIG. 2: $G^{(+)}(q)$ for $N = 2, L = 32$. For $T = 2.86 > T_{c2}$, $T = 2.76 \simeq T_{c2}$, and $T = 2.63 < T_{c2}$, $\lim_{q \rightarrow 0} G^{(+)}(q) \sim c(T)$, $\sim q$, and $\sim q^2$, respectively.

The vortex correlator for $N = 2$ is sampled in real space and $G^{(+)}(q)$ is found by discrete Fourier transformation, it is shown in Fig. (2). At $T = T_{c1}$ the leading behavior is $G^{(+)}(q) \sim q^2$ on both sides of T_{c1} . Consequently, due to (7), m_A and $m_{\Sigma h}$ are finite in this regime. This shows that the vortex tangle is incompressible and that the anomalous scaling dimension $\eta_A = 0$, which corresponds to a neutral fixed point. Below T_{c2} the dominant behavior is $G^{(+)}(q) \sim q^2$ whereas $G^{(+)}(q) \sim c(T)$ above T_{c2} . At $T = T_{c2}$, $G^{(+)}(q) \sim q$ indicating $\eta_A = 1$. Accordingly, m_A is finite below T_{c2} and zero for $T \geq T_{c2}$.

For $T \lesssim T_{c2}$, m_A scales according to $\mathcal{G}_A(q)^{-1}\frac{2}{\beta} = m_A^2 + Cq^{2-\eta_A} + \mathcal{O}(q^\delta)$ for small q where $\delta > 2 - \eta_A$ [16], with a corresponding Ansatz for $\mathcal{G}_{\Sigma h}(q)$. For each coupling we fit $\mathcal{G}_A(q)^{-1}$ data from system sizes $L = 8, 12, 20, 32$ to the Ansatz. The results for m_A (and $m_{\Sigma h}$, found similarly), are given in Fig. 3. The system exhibits Higgs mechanism at $T = T_{c2}$ when m_A drops to zero. Furthermore m_A has a kink at T_{c1} due to ordering of $\theta^{(1)}$. The anomalies in m_A coincide precisely with T_{c1} and T_{c2} determined from scaling of $\langle (S_V - \langle S_V \rangle)^2 \rangle$. Note also how $m_{\Sigma h}$ changes abruptly at T_{c2} . This is due to a sudden change in screening of $\sum_{\alpha=1}^N \mathbf{h}^{(\alpha)}$ by the vortex-loop proliferation at $T = T_{c2}$, giving an abrupt increase in $m_{\Sigma h}$, analogously to what happens for $N = 1, e \neq 0$ [11]. Above T_{c2} , A is massless, giving a compressible vor-

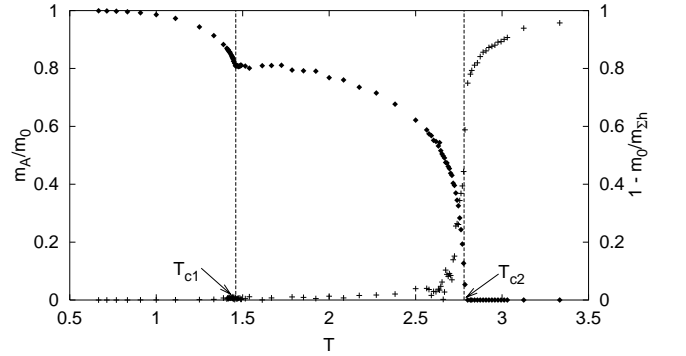


FIG. 3: The mass m_A (\blacklozenge) and $1 - m_0/m_{\Sigma h}$ ($+$) found from Eq. (7). Two non-analyticities can be seen in m_A at T_{c1} and T_{c2} , corresponding a neutral fixed point and a charged Higgs fixed point, respectively. An abrupt increase in $m_{\Sigma h}$ due to vortex condensation is located at T_{c2} .

tex tangle which accesses configurational entropy better than an incompressible one. Below T_{c2} , A is massive and merely renormalizes $|\Psi|^4$ terms in Eq. (1). The theory is effectively a $|\Psi|^4$ theory in this regime. Thus, the remaining proliferated vortex species originating in the matter fields with lower bare stiffnesses form vortex tangles as if they originated in a neutral superfluid. For the general N case, a Higgs mass is generated at the highest critical temperature, after which A merely renormalizes the $|\Psi|^4$ term, such that the Higgs fixed point is followed by $N - 1$ neutral fixed points as the temperature is lowered.

We now discuss the vortex mode $\mathbf{m}^{(1)} - \mathbf{m}^{(2)}$, demonstrating that it should be identified as a superfluid mode in the system. Its properties are controlled by $\mathcal{G}_{\Delta\mathbf{h}}(q) \equiv \langle |\mathbf{h}_q^{(1)} - \mathbf{h}_q^{(2)}|^2 \rangle$. A dual Higgs phenomenon for $N = 2, T = T_{c1}$ involving $\mathcal{G}_{\Delta\mathbf{h}}(q)$ may be demonstrated

$$\mathcal{G}_{\Delta\mathbf{h}}(q) = \frac{8\beta\lambda^{(1)}\lambda^{(2)}\psi^2}{|\mathbf{Q}_q|^2} \left\{ 1 - \frac{2\pi^2\beta\lambda^{(1)}\lambda^{(2)}\psi^2 G^{(-)}(q)}{|\mathbf{Q}_q|^2} - \frac{2\pi^2\beta(\lambda^{(1)} - \lambda^{(2)})G^{(m)}(q)}{|\mathbf{Q}_q|^2 + m_0^2} \right\} + (\lambda^{(1)} - \lambda^{(2)})^2 \mathcal{G}_{\Sigma\mathbf{h}}(q). \quad (8)$$

The $G^{(-)}(q)$ correlation function is always $\sim q^2, q \rightarrow 0$, but has a nonanalytic coefficient of q^2 , determined by the helicity modulus Υ of the neutral mode $\mathbf{m}^{(1)} - \mathbf{m}^{(2)}$. When Υ vanishes at T_{c1} through a disordering of $\theta^{(1)}$, thus destroying the superfluid neutral mode, the first and second term in the bracket cancel, which in turn cancels the $1/q^2$ term in $\mathcal{G}_{\Delta\mathbf{h}}(q)$. This produces a dual Higgs mass $m_{\Delta\mathbf{h}}$ defined by $\mathcal{G}_{\Delta\mathbf{h}}(q) \sim 1/(q^2 + m_{\Delta\mathbf{h}}^2)$ for $T > T_{c1}$. The remaining terms in Eq. (8) contribute to determining the actual value of $m_{\Delta\mathbf{h}}$. Thus, while $\mathbf{h}^{(1)} + \mathbf{h}^{(2)}$ is always massive, cf. Eq. (3), $\mathbf{h}^{(1)} - \mathbf{h}^{(2)}$ is massless below T_{c1} and massive above T_{c1} . Therefore $\mathbf{h}^{(1)} - \mathbf{h}^{(2)}$ plays the role of a gauge degree of freedom, providing a dual counterpart to \mathbf{A} in Eq. (1). This is evident when $|\psi^{(1)}|^2 = |\psi^{(2)}|^2$. Then Eq. (8) for $N = 2, e \neq 0$ has the same form as the dual gauge field correlator for the case $N = 1, e = 0$, which exhibits a dual Higgs phenomenon [11]. Thus, for $N = 2, e \neq 0$, $\mathbf{m}^{(1)} - \mathbf{m}^{(2)}$ behaves as vortices for $N = 1, e = 0$, *i.e. it is a superfluid mode arising out of superconducting condensates*. A nonzero $m_{\Delta\mathbf{h}}$ is produced by disordering $\theta^{(1)}$ at T_{c1} while a nonzero $m_{\mathbf{A}}$ is destroyed by disordering $\theta^{(2)}$ at T_{c2} .

We have analysed the N -color London model Eq. (2) in vortex representation Eqs. (4) and (5). The dual theory is given by Eqs. (3) and (6). For $N = 2$, we have performed large scale Monte Carlo simulations computing *i)* critical exponents α and ν , *ii)* gauge field and dual gauge field correlators, *iii)* the corresponding masses, and *iv)* critical couplings using FSS. For $\psi^{(1)} \neq \psi^{(2)}$ we find one *neutral* low-temperature critical point at T_{c1} , and one *charged* critical point at $T_{c2} > T_{c1}$. For general N , a Higgs mass $m_{\mathbf{A}}$ is generated at the highest critical temperature, followed by $N - 1$ neutral fixed points as the temperature is lowered.

These results apply to electronic and protonic condensates in liquid metallic hydrogen under extreme pressure. Estimates exist for T_{c2} for such systems, $T_{c2} \approx 160\text{K}$ [2], and hence $T_{c1} \approx 0.1\text{K}$. Hence, in addition to the emergence of the Meissner effect at T_{c2} and a corresponding divergence in the magnetic penetration length $\lambda \sim |1 - T/T_{c2}|^{-\nu/(2-\eta_{\mathbf{A}})}$ [17], there will also be a novel effect, namely a low-temperature anomaly in the mag-

netic penetration length $\lambda \sim 1/m_{\mathbf{A}}$ at T_{c1} , cf. Fig. (3), due to the appearance of superfluid modes arising from superconducting condensates.

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